In[7]:=
```
Clear[diffPercent];
diffPercent::usage =
  "Compute the relative difference between a 'sampleValue' and
    theoretical 'popValue' in per cent. The
    theoretical value '0' is treated specially.";
diffPercent[sampleValue_?NumberQ, popValue_?NumberQ] := Module[{result},
   If[popValue == 0, result = NumberForm[Abs[(sampleValue * 100)], {Infinity, 2}],
    result = NumberForm[Abs[(sampleValue - popValue) / 1 * 100], {Infinity, 2}]];
   Return[result]];
Clear[stdRandomVariable];
stdRandomVariable::usage =
  "Compute 'i' times 'j' radom variables of distribution 'distrib', returning
    'i' standardized random variables with each random variable a sum
    of 'j' random variables of the original distribution 'distrib'.";
stdRandomVariable[distrib_?DistributionParameterQ, i_Integer, j_Integer] :=
 Module[{Sij, Sn, Zn, betaMean, betaSD}, D = distrib;
  betaMean = Mean[D];
  betaSD = StandardDeviation[D];
  Sij = RandomVariate[D, {i, j}];
  Sn = Apply[Plus, Sij, 1];
  Zn = (Sn - j * betaMean) / (Sqrt[j] * betaSD);
  Return[Zn]]
```

Zentraler Grenzwertsatz

Wir illustrieren den zentralen Grenzwertsatz anhand der Beta-Verteilung.
i ist die Anzahl der summierten Zufallsvariablen S_n. Und j ist die Anzahl der ursprünglichen B-verteilten Zufallsvariablen.

In[13]:=
```
Clear[α, β, D, betaMean, betaSD];
α = 2;
β = 5;
D = BetaDistribution[α, β];
betaMean = Mean[D];
betaSD = StandardDeviation[D];
Grid[
 {{"B-Verteilung mit ", "α=" <> ToString[α] <> "und β=" <> ToString[β] <> "."},
  {"Mittelwert", "Standardabweichung"}, {betaMean, betaSD}}, Dividers → All]
```

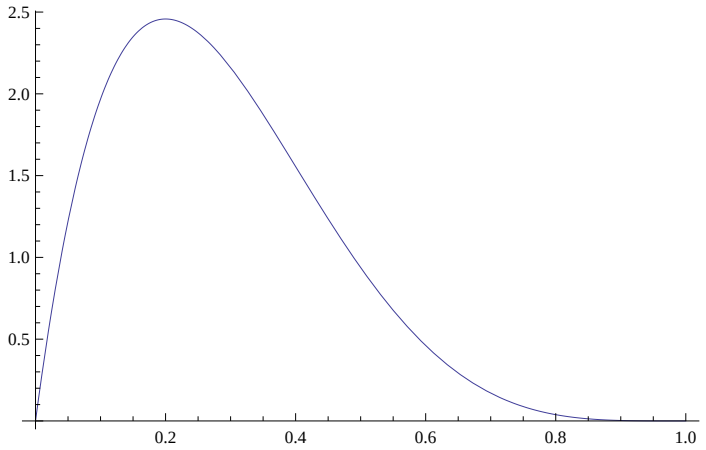
Out[19]=

B-Verteilung mit	α=2und β=5.
Mittelwert	Standardabweichung
$\frac{2}{7}$	$\frac{\sqrt{5}}{14}$

Wahrscheinlichkeitsfunktion der $B(\alpha, \beta)$-Verteilung mit α=2 und β=5.

In[20]:=
```
Plot[PDF[D, x], {x, 0, 1}]
```

Out[20]=



Die Level eines *Mathematica-Ausdrucks werden immer von der Wurzel weg gezählt, mit der Wurzel = Level 0.*

In[21]:=
```
Clear[i, j, Sij, Sn, Zn];
i = 100;
j = 1000;
Sij = RandomVariate[D, {i, j} ];
Sn = Apply[Plus, Sij, 1]; (* Choose the right level *)
Zn = (Sn - j * betaMean) / (Sqrt[j] * betaSD);
```

Wir erwarten eine Standardnormalverteilung dr Z_n, also μ=0 und σ=1.

In[27]:=
```
Grid[{{"Mittelwert Zn", "Standardabweichung Zn"},
  {Mean[Zn], StandardDeviation[Zn]}}, Dividers -> All]
```

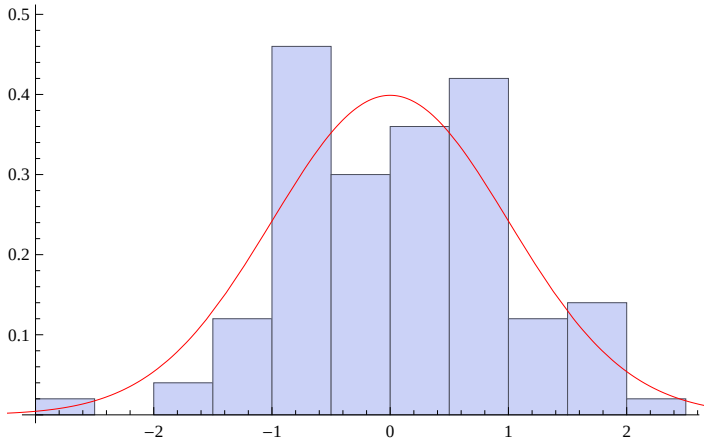
Out[27]=

Mittelwert Z_n	Standardabweichung Z_n
0.0815114	0.932035

Hier der Vergleich der Stichprobenverteilung mit der theoretischen Verteilung.

In[28]:=
```
Clear[betaHist];
betaHist = Histogram[Zn, Automatic, "PDF", Epilog →
   First@Plot[PDF[NormalDistribution[0, 1], x], {x, -4, 4}, PlotStyle → Red]]
```

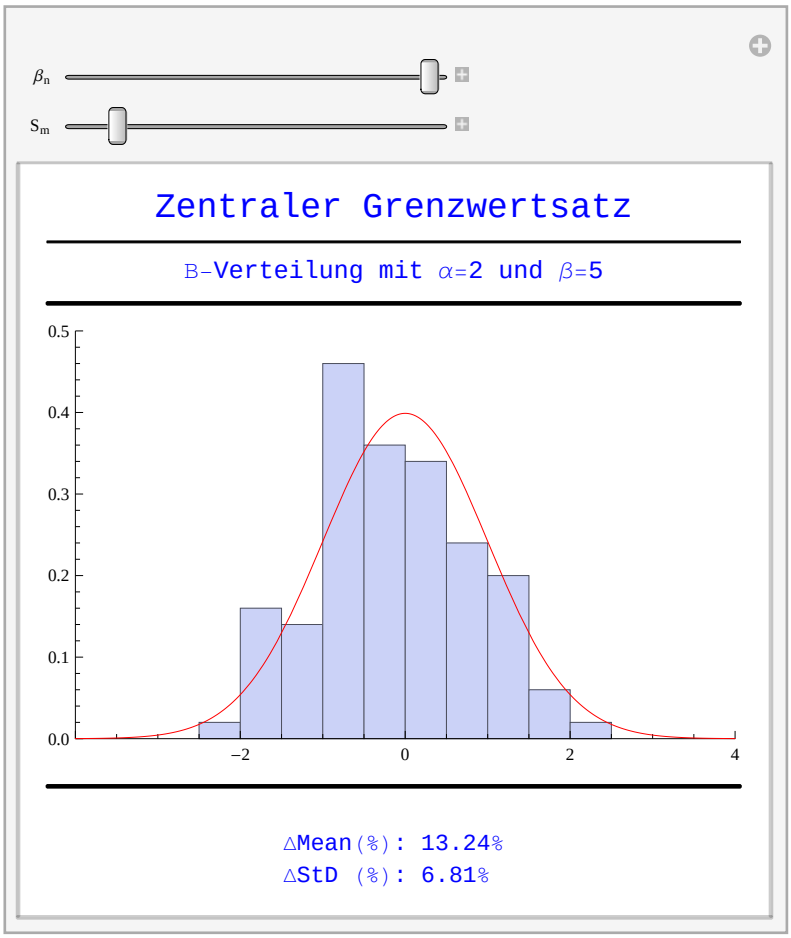
Out[29]=



Den Konvergenz-Effekt kann man schön sehen, wenn man die Anzahl der S_n variert.

In[30]:=
```
cltDemo = Manipulate[Module[{Zn, meanSample, sdSample},
    Zn = stdRandomVariable[BetaDistribution[2, 5], i, j];
    meanSample = Mean[Zn];
    sdSample = StandardDeviation[Zn];
    Column[{Style["Zentraler Grenzwertsatz", Larger, Blue, 18],
      Style["B-Verteilung mit α=2 und β=5", Blue],
      Histogram[Zn, Automatic, "PDF", PlotRange → {{-4, 4}, {0, 0.5}},
       AxesOrigin → {-4, 0}, Epilog → First@Plot[PDF[NormalDistribution[0, 1], x],
          {x, -4, 4}, PlotStyle → Red], ImageSize → Medium],
      Style[Column[{"\nΔMean(%): " <> ToString[diffPercent[meanSample, 0]] <> "%",
         "ΔStD (%): " <> ToString[diffPercent[sdSample, 1]] <> "%"}], Blue, 12]},
     Dividers → Center, Spacings → {0, 1.5}, Alignment → Center]],
   {{j, 100, "βn"}, 1, 1000, 1},
   {{i, 100, "Sm"}, 1, 1000, 1},
   SaveDefinitions → True]
```

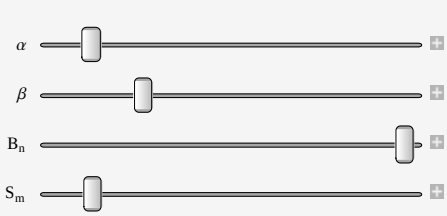
Out[30]=



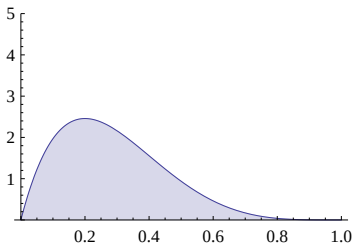
Dass es nicht an der Parametrisierung der Ausgangsverteilung liegt, zeigt diese Demo. Hier kann man auch die Parameter α und β varieren.

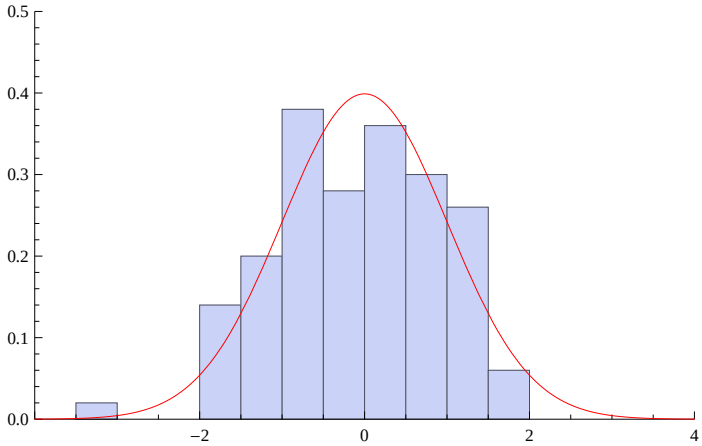
In[31]:=
```
cltDemo1 = Manipulate[Module[{Zn, meanSample, sdSample},
    Zn = stdRandomVariable[BetaDistribution[α, β], i, j];
    meanSample = Mean[Zn];
    sdSample = StandardDeviation[Zn];
    Pane[Column[{Style["Zentraler Grenzwertsatz", Larger, Blue, 18],
       Grid[{{Style["B_{α,β}(x)", Blue], Style["Numerisch", Blue]},
         {Plot[PDF[BetaDistribution[α, β], x], {x, 0, 1}, Filling → Bottom,
           PlotRange → {0, 5}], Pane[PDF[BetaDistribution[α, β], x] // First //
            First, {200, 175}, Alignment → Center]}}, Dividers → Center],
       Histogram[Zn, Automatic, "PDF", PlotRange → {{-4, 4}, {0, 0.5}},
        AxesOrigin → {-4, 0}, Epilog → First@Plot[PDF[NormalDistribution[0, 1],
            x], {x, -4, 4}, PlotStyle → Red], ImageSize → Medium], Style[
        Column[{"\nΔMean(%): " <> ToString[diffPercent[meanSample, 0]] <> "%",
          "ΔStD (%): " <> ToString[diffPercent[sdSample, 1]] <> "%"}], Blue, 12]},
      Dividers → Center, Spacings → {0, 1.5}, Alignment → Center], {400, 550}]],
   {{α, 2, "α"}, 0.1, 20, 0.1}, {{β, 5, "β"},
    0.1,
    20,
    0.1}, {{j, 100, "B_n"},
    1,
    1000,
    1}, {{i, 100, "S_m"},
    2,
    1000,
    1}, SaveDefinitions →
    True]
```

Out[31]=



Zentraler Grenzwertsatz

$B_{\alpha,\beta}(x)$	Numerisch
	$\{30\,(1-x)^4\,x,\ 0<x<1\}$



ΔMean (%): 8.53%
ΔStD (%): 4.23%